**Problem 1:**

1. E(X) = (E(x1) + E(x2) + … + E(xi) + … + E(xn)) for all xi tosses

For any toss, xi, E(xi) = 1/9 + 2/3 + 3/9 + 4/6 + 5/9 + 6/6 = 10/3

So, E(X) = n\*10/3 = **10n/3**

1. E(X) = n \* (probability the dice is even)

= n \* (1/3 + 1/6 + 1/6) = n \* 2/3

= **2n/3**

**Problem 2:**

For each j, where j is an element of 1 … n, there are n possible values for f(j), and each is equally likely (since it is uniform). Thus. P(f(j) = i) = 1/n for any j, i elements of S

For each i, P(i has one inverse with respect to f) = P(f(j) = i) \*  ∏P(k != i) for any k element of S-j

P(k != i) = 1 – P(k = i) = (n-1)/n

So, for each I, P(I has one inverse with respect to f) = 1/n \* ((n-1)/n)n

E(|U|) = n \* (1/n \* ((n-1)/n)n) = **((n-1)/n)n**

For each j, where j is an element of 1 … n, there are n possible values for f(j), and each is equally likely (since it is uniform). Thus. P(f(j) = j) = 1/n

E(|V|) = n \* (1 \* (1/n)) = n/n = **1**

**Problem 3:**

For each possible element of Ch, E(xi) = 1/2n (Since E(X) = ½ = n \* 1/2n)

Var(xi) = ((-1/2n)2 + (1 – 1/2n)2 ) / 2 = (1/4n2 + 1 – 2/2n + 1/4n2) / 2 = 1/4n2 + 1/2 + 1/2n

Var(X) = n \* 1/4n2 + 1/2 + 1/2n = 1/4n + n/2 + 1/2

{(|Ch| >= 1) = P(|Ch| - ½ > ½)

<= Var(X) / (1/4) = 1/n + 2n + 2

**Problem 4:**

For any dice roll x, E(x) = 3.5, and for n dice rolls, E(X) = 3.5n

Var(x) = 105/36, Var(X) = 105n/36

Also note, P(X <= 2n) = P(X >= 5n)

Markov: P(X >=5n) <= E(X)/5n = 3.5/5 = .7

**So, P(X <= 2n) <= .7**

Chebychev: P(X >= 5n) = P(X – 3.5n >= 1.5n)

<= P(|X – 3.5n| >= 1.5n) = P(|X – E(X)| >= 1.5n)

<= Var(X) / (1.5n)2 = 105n / (81n2) = 105/81n

**So, P(X <= 2n) <= 105 / 81n**

Chernoff: P(X >= 5n) = P(X – 3.5n >= 1.5n)

**<=** P(|X – 3.5n| >= 1.5n) = P(|X/n – 3.5| >= 1.5) = P(|X/n – 3.5| >= 3.5 \* (7/3))

<= 2e ^ (-(7/3)2\*n\*3.5/2) = 2e ^ (-343n/36) = 2e ^ (-9.52778n)

**So, P(X <= 2n) <= 2e ^ (-9.52778n)**

**Problem 5:**

Text